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**Measuring Autonomy Structures in a Montessori Classroom:
Analysis of Patterns in Time and Configuration**

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Abstract

Research studies have found that autonomy-support has a positive impact on perceived competence and intrinsic motivation of students. Montessori classrooms are characterized by an emphasis on autonomy-support through the provision of student choice over learning activities. This case study describes the structural configurations of a Montessori classroom using nine properties from a logico-mathematical general systems model (Axiomatic Theories of Intentional Systems). It was found that autonomy supportive strategies employed by Montessori teachers encouraged the formation of *Complete Connectivity* and *Interdependence* in terms of *Instructional* and *Support* affect relations. This in turn was associated with student affect and intrinsic motivation for learning. The applicability of system properties for characterizing classroom structure is discussed with respect to Analysis of Patterns in Time and Configuration.

Deci, Vallerand, Pelletier & Ryan (1991) defined autonomy or self-determination as a state where one's volition was totally internalized and unaffected by external conditions. Self-determination theory proposes that autonomy-support in social environments is essential for fostering intrinsic motivation. A distinguishing feature of Montessori classrooms is its philosophy to educate children towards self-mastery and independence (Montessori, 1964). When compared to traditional middle schools, Montessori students have been found to have experienced more teacher support and psychological safety (Rathunde & Csikszentmihalyi, 2005).

The present qualitative case study analyzes classroom structures that supported student autonomy in a Montessori classroom. Using the Axiomatic Theories of Intentional Systems (ATIS) as a theoretical model, this study also explores how a logico-mathematical general system model can be used to characterize the configurations underlying classroom interaction.

Montessori Classrooms and Autonomy-Support

Self-determination theory defines the need for autonomy or self-governance as one of the three basic human needs (Deci *et al.*, 1991). Surveys of school-age children using large-sample sizes of at least 300 have found that students perceived teachers to be highly autonomy-supportive when they gave choice, had confidence in their ability, respected and empathized with them (Hardre & Reeve, 2003). Comparing between different types of autonomy-supportive strategies, Assor Kaplan and Roth (2002) found that students developed better affect and cognitive engagement when teachers fostered relevance, and suppressed criticism. On the other hand, Assor, Kaplan, Kanat-Maymon, and Roth (2005) found that students perceived teachers to be controlling when they gave frequent directives, interfered with students' preferred pace of learning, and disallowed independent opinions.

Experimental studies also investigated the impact of autonomy-support by controlling for variables such as the degree of choice, the extent of directives applied, and whether a rationale was provided for task engagement (Grolnick & Ryan, 1987; Joussemet, Koestner, Lokes, & Houliort, 2004). These studies found that social contexts that were autonomy-supportive had positive effects on children's interest for learning, level of conceptual learning, and willingness to perform uninteresting activities. Structural equation models developed by Valas and Sovik (1993), Hardre and Reeve (2003), and Standage, Duda and Ntoumanis (2005) also showed that perceived autonomy-support in the classroom significantly predicted higher intrinsic motivation and perceived competence in students. On the other hand, Ntoumanis (2002) found that *self-determined* students tend to exhibit greater effort and cooperative learning. The few qualitative studies conducted found that teachers' motivation styles resulted in substantially different classroom climates (Manouchehri, 2004). Their perception of student ability and prior knowledge also affected the extent to which they allowed student choice (Flowerday & Schraw, 2000).

Montessori schools exhibit the characteristics of autonomy-support as described by the preceding studies. Its pedagogy resulted from Maria Montessori's observations, as a physician and as an anthropologist, of school children in the San Lorenzo slums of Rome. She believed that education should help each child become a disciplined individual who is "master of himself, and can, therefore, regulate his own conduct when it shall be necessary to follow some rule of life." (Montessori, 1964, p. 86). A distinctive feature of Montessori teaching is that teachers direct and support students to correct their own errors instead of providing direct answers to them (Cossentino, 2005).

Montessori (1964) also believed in a scientific approach where children were treated as natural phenomenon to be observed and understood. Teachers prepare the classroom with *Works* or instructional activities, and allow students choice to engage in what interests them (Cossentino, 2006). They then make observations, and modify the *Works* to maintain students' learning engagement, and to stimulate interest in the *Works* they do not naturally choose. Even though children were given liberty to manifest themselves naturally, Montessori believed that they should be disciplined when they cause disruptions to learning. This is necessary for teaching them to discern between good and evil. However, she also argued that discipline was not used to create quiet and inactive children; emphasizing that one should not confound "good with immobility, and evil with activity." (Montessori, 1964, p. 93) A child who is disciplined should be in a state of normalization where "the child works freely and happily with the materials supplied to it, at a pace suiting its own nature, without rewards or punishments, or being spurred unduly by competition or compelled by the teacher." (Lubienski Wentworth, 1999, p. 15).

When the social context of Montessori and traditional middle schools were compared, Rathunde and Csikszentmihalyi (2005) found that Montessori students reported more support from teachers; more order in the classroom, and a greater feeling of emotional/psychological safety. They also spent more time with academic work, group work, collaborative learning, and individual projects; but less time in passive listening via lecturing and note-taking. The authors concluded that analysis of school contexts that fostered intrinsic motivation could provide concrete ideas for improving student engagement in public schools. However, current studies of intrinsic motivation of public schools have yet to provide conclusive advice for implementing autonomy-supportive systems in classrooms (Urda & Turner, 2005). An analysis of Montessori classrooms could inform this gap.

Logico-Mathematical General System Theory Models

Ludwig von Bertalanffy (1968) conceptualized general systems theory with the postulation that systems could have common characteristics and behavior regardless of whether these were scientific, natural or social. At that time, general systems theory (GST) was viewed as a way to conceptually unify increasingly specialized scientific disciplines (Boulding, 1956). One approach used to formalize GST was through the development of mathematical theory models to describe systems behavior and characteristics (Mesarović, 1964; Maccia & Maccia, 1966).

In particular, Maccia and Maccia's SIGGS theory model was developed for educational theorizing. It proposed that the general characteristics and behavior underlying educational systems could be defined by studying the *affect relations* or the characteristics of connections between system components. An example of a system typology was an Instructional Unit that consisted of components such as teachers, students and teaching devices. Mathematical precision in SIGGS was achieved by integrating set (S), information (I), digraph (G) theories with general systems theory (GS). Set theory provided the mathematical basis for defining system components and boundaries. Digraph theory characterized the direction of connections between points, while Information Theory described how components selected and interpreted available information from each other. Seventy-three system properties were developed that described *affect relations* and the predicate calculus was then used to formalize these properties quantitatively (Maccia & Maccia, 1976).

General systems theory has also been formalized through the development of guiding principles for applying "systems thinking" to organizational problems (Ackoff, 1981; Senge, 1990; Hammer & Champy, 2001). However, it has been difficult to describe system levels and

boundaries with precision when this approach is applied to social sciences (Kast & Rosenzweig, 1972). Therefore, one advantage of mathematical models is the presence of an “exact language permitting rigorous deductions and confirmation (or refusal) of theory” (von Bertalanffy, 1972, p.30). In the SIGGS theory model for example, the use of mathematical language facilitated the application of logic towards study of relationships between system properties. These relationships were expressed as 201 hypotheses that characterized the behavior of school systems.

While the SIGGS theory model was comprehensive, it lacked consistent nomenclature. Thompson (2005a; 2005b; 2005c; 2006; 2007) formulated the Axiomatic Theories of Intentional Systems (ATIS) by improving on the consistency of SIGGS nomenclature, using predicate calculus to convert SIGGS hypotheses into a set of axioms or assumptions underlying general systems behavior, and by deducing additional theorems. The development of ATIS extended the 73 SIGGS properties into 136 basic, structural and dynamic system properties. ATIS basic properties describe system characteristics such as the number of system components (*size*), and the number of affect relations or connections between system components (*complexity*). These properties provide descriptive information about the characteristics of the system. Structural properties characterize the nature and strength of connections between system components with respect to an *affect relation* while dynamic properties describe changes that occur as a result of changes within a system, or its interactions with the *negasystem* (environment outside system boundaries).

Hug and King (1984) proposed that general system properties such as “integration” and “stability” could be used to understand reciprocal interaction between components in classroom systems. Despite their potential use, applications of general systems properties have rarely progressed beyond conceptual discussions in current education literature. In addition, the

majority of the motivation studies have been either experimental studies or surveys. Urdan and Turner (2005) commented on the limitations of transferring these results to actual classrooms where motivational influences may not necessarily be unidirectional from teachers to students, and raised the need for more qualitative studies. Logico-mathematical formulations of general systems theory properties could be used to support qualitative analysis by modelling and measuring the configuration of motivational influences in classroom systems.

Cossentino (2005) found Montessori classrooms to be characterized by discernable rituals that teachers use. Using the ATIS theory model, this study attempted to describe these rituals by measuring nine general system properties from the interactions between teachers and students. The properties measured were: *Centrality, Complexness, Complete Connectivity, Heterarchical Orderness, Hierarchical Orderness, Interdependence, Passive Dependence, Size and Strongness*. These measurements were also triangulated with qualitative and survey data to identify the structural configurations that support student autonomy in a Montessori classroom.

Method

Subjects

The study was conducted in an upper elementary (fourth to sixth grade) classroom of a Montessori school located in the state of Indiana. It consisted of 28 students, ranging from 9 to 11 years old, a Montessori-certified head teacher, and two assistant teachers.

Classroom Context

In each 16-week session, students needed to complete a mandatory number of *Works* which include research projects in Physical Science, Natural Science, History, Geography; book reports where topics were collaboratively chosen by teachers, parents, and students; and

workbooks in Math and flash card drills. Teachers made a punch on individual cards that tracked their progress when an item of their required *Works* was completed.

A typical day in the school was from 8:30 a.m. to 3:30 p.m. The morning started with students working on a *Head Problems* worksheet that consisted of math and logic-related problems designed by teachers. When this was completed, students moved into the *Morning Work Period* where they were free to work individually or collaboratively on their *Works*. A short intermission occurred at about 9:40 a.m. when two students responsible for making the day's bread started distributing it. Students usually took this time to eat and socialize before resuming their work while those assigned to make the next day's bread would start working in the kitchen. Teachers also used the *Morning Work Period* to engage students in individual feedback sessions where improvements to their projects drafts were discussed. About half an hour before recess at noon, students re-gathered to present their answers for the *Head Problems*. After lunch at 1 p.m., students attended Spanish or History classes and would end the day with cleanup duties at about 3 p.m.

Procedure

This study was first approved by the School Board of the Bloomington Montessori Association, following which consent forms for study participation were circulated to parents. Consent was subsequently obtained for 10 students. Ten one-hour observations were then conducted in the classroom during April 2006 where two to three observations were made each week. Each observation was carried out for about an hour, usually between 8:30 a.m. to 9:45 a.m. This time segment was chosen upon consultation with teachers as they felt that the classroom activities best reflected how the major portion of students' learning goals were completed in a mixed-grade Montessori classroom. Even though the *Morning Work Period* officially started at

9:30 a.m., students typically completed *Head Problems* early and were well into the *Morning Work Period* by 9:15 a.m. It was not necessary to observe the entire *Morning Work Period* as students tend to settle down to a consistent work pattern by about 9:45 a.m. The researcher adopted a non-participatory role during observations.

One challenge faced when collecting detailed interaction data in a Montessori classroom was its emphasis on children's freedom to move around and work where they felt most comfortable. While the use of video equipment would have afforded greater precision for data collection, it was not possible to film only those subjects with parental consent without disrupting the normal operations of the classroom. Therefore, ethnographic field-notes were used to record the various types of interactions occurring between teachers and students. When studying work patterns in a Montessori classroom, Cossentino (2006) employed the same data collection strategy in an effort to respect its classroom norms. The lack of recording devices did not limit her ability to make verbatim records of many interactions because students tend to develop deep engagement *Works*, and extended speech events were reported to be rare.

Before actual data collection began, an informal observation was carried out to understand the dynamics of student-teacher interaction in a Montessori classroom. Like Cossentino (2006), it was observed that movement and interactions in the classroom tend to be more pronounced during the five to ten minutes when students were transitioning between *Head Problems* and the *Morning Work Period*. Activity patterns tend to be relatively stable once they began working on the learning activities. Despite this, the following measures were used to improve the consistency and comprehensiveness of raw data collected to describe general classroom activities:

1. By studying the classroom map provided by the teachers, 16 key work areas were indentified.

Fifteen of them were clusters of teacher and student work desks, while the remaining was the

computer cluster. Each work area was assigned a running number starting from the left to right side of the classroom. Following this, a data collection sheet was formulated to systematize the recording of interactions by work area.

2. Every five minutes, a visual scan of each work area was done, following which the number of children, number of teachers, and the activity happening at that work area was recorded on the data collection sheet. In a typical hour of observation, 12 data sheets would be filled out, one for each five minute interval.

In addition, it was found that data recording could be speeded up by using abbreviations for common activities. Using the constant comparative method (Cresswell, 1998), these abbreviations were gradually developed and used in data recording. For example, “A” denoted working individually on assignment, “C” denoted chatting and “S” denoted students who were giving each other peer support to complete their assignment. These abbreviations helped to shorten the time required to record data during the routine visual scan. Across time, there were increased opportunities to move around the room and collect anecdotal data related to:

1. How teachers executed teaching strategies.
2. How teachers communicated motivational or control behaviors such as giving choice, encouraging student questions, listening to students’ point of view or use of discipline and control.
3. How teachers and peers supported learning.
4. Social activities or topics that teachers and students engaged in.
5. How classroom resources were used.

Only quotations from students with parental consent are cited in this report.

To determine if the motivational style of teachers is associated with their teaching method, the *Problems in Schools Questionnaire* (SDT Website, 2006a) was administered to each teacher. To avoid bias to teachers' instructional behaviors, the purpose of the questionnaire and their results were not revealed until the end of observations. A hardcopy of the questionnaire was given to them at the beginning of the study and collected at the end of the study. It consisted of eight vignettes that described typical motivational problems exhibited by elementary-age students such as not completing assignments, agitating other children, failing in tests, and problems with being accepted by other children. Four responses were available for each vignette and each operationalized an item in the highly controlling (HC), moderately controlling (MC), moderately autonomy supportive (MA), and highly autonomy supportive (HA) subscales. Teachers rated the 32 possible responses on a Likert scale of 1 (very inappropriate) to 7 (very appropriate). The motivational style of each teacher was obtained by comparing their total score for the eight questions associated with each subscale. The higher the score obtained for a subscale, the more dominating that motivational style. This scale was used with elementary school teachers in Deci, Schwartz, Sheinman and Ryan (1981), Reeve, Bolt and Cai (1999) and Cai and Robinson (2002).

The impact of instructional activities on students' level of intrinsic motivation with respect to doing schoolwork was assessed through the *Academic Self-Regulation Questionnaire* (SDT Website, 2006b). This was administered only to the students with parental consent. It was also administered both at the beginning and end of the observations so that consistency of ratings could be verified. This questionnaire consisted of 32 questions that measured four sub-scales characterizing children's motivation for doing schoolwork: External Regulation (ER), Introjected Regulation (IR), Identified Regulation (IDR) and Intrinsic Motivation (IM). Each question was

scored on a Likert Scale of 1 (Not at all true) to 4 (Very true). The sub-scale with the highest average score characterized their predominant motivational tendency. This scale has been used with elementary school children in studies conducted by Grolnick and Ryan (1987), Grolnick, Ryan and Deci (1991), Patrick, Skinner and Connell (1993) and Miserandino (1996).

A one-hour semi-structured focus-group interview was also conducted with the teachers at the end of the observations. The following were open-ended questions used in the interview protocol:

- What were your strategies for handling student questions when they asked for help?
- How did you balance between autonomy and control in the classroom? When was control necessary and when was autonomy better?
- What role did rewards play in your classroom? Did you use extrinsic rewards? What were some examples of intrinsic rewards you have used?
- How would you interpret the results from the students' survey?

Handwritten interview notes were taken during the interview. Typewritten transcripts were subsequently sent to the teachers for verification via e-mail. One teacher responded by making additional comments and clarifications on a soft-copy of the transcript, precluding the need for re-verification.

Data Analysis

Categorizing classroom interactions

The constant comparative method (Creswell, 1998) was used to derive categories of interactions from observation data. A list of 43 common interactions was found to occur between teachers and students, students and resources such as computers, books etc, and between students. These were further grouped into three categories: *Instructional*, *Support* and *Control*. An

interview was held with the teachers to verify the interactions and their proposed categories. Teachers agreed that these adequately captured the types of interactions they experienced in their classroom and did not suggest any amendments. Motivational structures underlying the Montessori system results from the various interactions students experience through learning activities. The three categories were therefore regarded as the types of affect relations underlying the interactions in this classroom. They describe connections between system components whose presence could be qualified by observable interactions.

Measuring system properties

In order to measure system properties, interactions were converted into ordered pairs characterizing the direction of connections between components. This is the notational system used by the Set and Diagraph theories which underlie ATIS. For this classroom system, a coding system of t_1 , t_2 , and t_3 was used to denote each teacher, and a running number of s_1 to s_{28} denoted each student. Codes were also created for assignments such as *Head Problems*, student-specific *Works*, and resources such as computers and lab apparatus where applicable.

Interactions underlying each affect relation was identified by matching of frequency and pattern occurrences (Yin, 2003), and then described as ordered pairs. For example, if student 1 asked and received help on a Math problem from teacher 2, it was coded as the simultaneous presence of the following ordered pairs with respect to an *Instructional* affect relation: (s_1, t_2) , (t_2, s_1) . The first denoted an instructional request initiated by the student to the teacher while the second denoted instructional content being transmitted from the teacher to the student. This interaction can also be represented by a matrix (Figure 1), or by a digraph (Thompson, 2006).

The same ordered pairs could also describe an interaction whereby student 1 asked and received help from teacher 2 to scan a diagram for a Science paper. However, these ordered pairs

would be associated with a *Support* affect relation as they are not related to the direct transmission of instructional content, but are interactions needed to support completion of learning activities. There is a need to distinguish between the types of affect relations because general system properties in ATIS are computed with respect to specific *affect relations*.

System properties for each type of affect relation were then derived by calculating the number of connections between ordered pairs that existed according to the characteristic of a property. The nine properties measured in this study were formulated by Thompson (2006) in predicate calculus as follows:

1. Size

Size, $Z(\mathfrak{S})$, =_{df} components of an object-set.

$$Z(\mathfrak{S}) =_{df} \exists \omega \subset \mathfrak{S}_0$$

Size is a basic property measuring the number of components in a system. If only the 3 teachers and 28 students were considered as system components, the *Size* of this classroom will be 31. However, *Size* could also include computers, books, or specific *Works* that form pertinent affect relations with teachers and students. Each of these resources increase the *Size* of this classroom system.

2. Complexity

Complexity-component partition, ${}_x\mathfrak{S}$, =_{df} a *partition*, $\mathcal{Q}_y = (\forall \subset \mathcal{G}_0, \mathcal{R} \subset \mathcal{G}_x)$, characterized by the number of affect-relations.

$${}_x\mathfrak{S} =_{df} |\mathcal{Q}_y(\mathcal{R})|$$

Complexity is another basic property measuring the total number of connections occurring between components in a system. If the only interaction in the system was $\{(s1,t2), (t2,s1)\}$, *Complexity* will be 2 as there is one connection initiated by s1 to t2, and another from t2 to s1.

3. Passive Dependence

\mathcal{M} : **Passive dependent-component partition measure**, $\mathcal{M}(\mathcal{S}_{PD})$, =_{df} a measure of receiving component affect-relations.

$$\mathcal{M}(\mathcal{S}_{AD}) =_{df} [(\sum_{i=1, \dots, n} [\prod d_T(v) \div \log_2 |\mathcal{A}_i|]) \div n] \times 100$$

Passive Dependence is a structural property that measures the extent to which system components are receiving connections. It is computed by first tabulating the number of connections received by each component, and then taking their product. For the affect relation set: $\{(s1, t2), (t2, s1)\}$, both s1 and t2 receive one connection. Their product is 1. This figure is then compared with the total path permutations that are possible between the six components ($|\mathcal{A}_i|$), which is 2. As the total number of path permutations will become very large when more system components are being analyzed, a scaling factor is introduced by applying the logarithmic function to $|\mathcal{A}_i|$, and then multiplying it by 100. This derives a *passive dependence* measure of 100. The n refers to the number of types of affect relations being analyzed. If more than one type of affect relation is being analyzed, an average of *active dependence* across each affect relation needs to be considered. For this analysis, n is taken to be 1 as affect relations are analyzed separately.

4. Interdependence

\mathcal{M} : **Interdependent system measure**, $\mathcal{M}(\mathcal{S}_N)$, =_{df} a measure of initiating and receiving associated component affect-relations.

$$\mathcal{M}(\mathcal{S}_N) =_{df} [(\sum_{i=1, \dots, n} [\log_2 \prod |r_{d(I,T)}(e) = 1|] \div \log_2 |\mathcal{A}_i|)] \div n] \times 100$$

In contrast with *passive dependence* that measures the extent of receiving connections, *Interdependence* measures the extent to which components initiate and receive connections. The *Interdependent* paths are computed by first identifying components that both initiate and receive connections. Then, the product of total paths initiated and received by these components is computed. For the affect relation set: $\{(s1, t2), (t2, s1)\}$, both s1 and t2 initiate

and receive two connections. Their product is 4. After applying the scaling factor to the $|A_i|$ of 2, an *Interdependence* measure of 200 is obtained.

5. Strongness

\mathcal{M} : **Strong-component partition measure**, $\mathcal{M}(\mathcal{S})$, =_{df} a measure of the degree of connected components.

$$\mathcal{M}(\mathcal{S}) =_{df} [(\sum_{i=1, \dots, n} [\log_2 \prod d(v) \div \log_2 |A_i|]) \div n] \times 100$$

Strongness measures the extent to which system components are connected. It is measured by the product of total connections received and initiated by each component. For the affect relation set: $\{(s1,t2), (t2,s1)\}$, both s1 and t2 have two connections each. Their product is 4. After applying the scaling factor to the $|A_i|$ of 2, an *Interdependence* measure of 200 is obtained. In this simple case, *Strongness* has the same magnitude as *Interdependence* as both s1 and t2 initiate and receive connections. When some components do not simultaneously initiate and receive connections, *Strongness* and *Interdependence* will differ.

6. Centrality

\mathcal{M} : **Centralized-component partition measure**, $\mathcal{M}(\mathcal{C})$, =_{df} a measure of primary-initiating, non-adjacent component affect-relations.

$$\mathcal{M}(\mathcal{C}) =_{df} [(\sum_{i=1, \dots, n} [r_{d(\mathcal{C})}(e) > 1]) \div \log_2 |A_i|] \div n] \times 100$$

This property measures the extent to which primary-initiating components have indirect control. It is computed by the total path length of connections from primary-initiating components that have a path length greater than 1 and comparing it $|A_i|$. Primary-initiating components are those that only initiate and do not receive connections. For the affect relation set: $\{(s1,t2), (t2,s1)\}$, Centrality is 0 as there are no primary-initiating components since both s1 and t2 initiate and receive connections.

7. Complete Connectivity

\mathcal{M} : **Complete-component partition measure**, $\mathcal{M}_{(CC)\mathfrak{S}}$, =_{df} a measure of pair-wise directed associated component affect-relations.

$$\mathcal{M}_{(CC)\mathfrak{S}} =_{df} [(\sum_{i=1}^n [r_{d(\epsilon)} \geq 1]) \div \log_2 |\mathcal{A}_i|] \div n \times 100$$

Complete connectivity measures the extent to which system components are able to connect to other components either directly or indirectly. This is computed by the sum of completely connected paths occurring in the system. For the affect relation set: $\{(s1,t2), (t2,s1)\}$, both $s1$ and $t2$ have one direct connection to each other, resulting in 2 completely connected paths. After comparison to $|A_i|$ and the applying the scaling factor, a *Complete connectivity* measure of 200 is obtained.

8. Hierarchical Orderness

\mathcal{M} : **Hierarchy system measure**, $\mathcal{M}_{(HO)\mathfrak{S}}$, =_{df} a measure of a *tree*.

$$\mathcal{M}_{(HO)\mathfrak{S}} =_{df} [(\sum_{i=1}^n [(|r_{d(branch)}(\epsilon) \geq 1| \times |\max(r_{d(walk)}(\epsilon) + 1)|]) \div \log_2 |\mathcal{A}_i|] \div n \times 100$$

Hierarchical orderness measures the extent of the occurrence of a *tree*. A tree is an acyclic simple-graph. Except for the root, every connected component is directly connected to only one other component. The root is an initiating component (does not receive) and is directly connected to one or more other components. For the affect relation set: $\{(s1,t2), (t2,s1)\}$, there is no hierarchical order, and hence the measure is zero.

9. Heterarchical Orderness

\mathcal{M} : **Heterarchy system measure**, $\mathcal{M}_{(HA)\mathfrak{S}}$, =_{df} a measure of initiating and receiving associated component affect-relations, or receiving associated (leaf) component affect-relations.

$$\mathcal{M}_{(HA)\mathfrak{S}} =_{df} [(\sum_{i=1}^n [(|r_{d(LT)}(\epsilon) \geq 1| + |r_{d(leaf)}(\epsilon) \geq 1|]) \div \log_2 |\mathcal{A}_i|] \div n \times 100$$

A pair of components is associated if they are either adjacent or non-adjacent and have a directed connection between them. In other words, the length of the path between associated components is greater than or equal to one. Components are heterarchy-connected if they are

associated and each associated pair has a two-way connection, or if a component is a leaf (receiving only) and associated. For the affect relation set: $\{(s1,t2), (t2,s1)\}$, the measure of heterarchical orderness is 200.

For purposes of brevity, the definitions and examples are quite simple here. The interested reader can view definitions and examples of these and other terms and properties in the ATIS Glossary at: <http://www.indiana.edu/~aptac/glossary/> (Thompson, 2007).

Results

Structural Differences Arising from Choice

The activity pattern of *Head Problems* followed by the long *Morning Work Period* was relatively stable across the ten observation days. Students had to complete the worksheets prescribed by teachers during *Head Problems* but could choose the *Works* they wanted to do, and whether they wanted to work on them individually or collaboratively during the *Morning Work Period*. The different levels of student autonomy had an impact on the system's structural configurations with respect to *Choice* (See Figure 2).

Complexity was substantially lower during *Head Problems* as only one *Choice* affect relation was present; being that between the teacher and the problems assigned to students. This is represented as an affect relation set, where a100 denotes the *Head Problems* for that particular day:

$$\{t1, a100\}$$

Each day, a teacher will prepare specific problems for students to solve. In comparison, the *Morning Work Period* found *Choice* affect relations occurring between each student and the

Works they selected, thereby resulting in a higher level of *Complexity*. This is modeled by the affect relation set:

{(s1,a1) (s2,a2) (s3,a3) (s4,a4) (s5,a5), (s6,a25), (s7, a26), (s8,a29), (s9,a6) (s10,a7)
 (s11,a8) (s12,a9) (s13,a10) (s14,a11) (s15,a12) (s16,a13) (s17,a14) (s18,a15) (s19,a16) (s20,a17)
 (s21,a18) (s22,a19) (s24,a20) (s25,a21) (s26,a22) (s27,a27) (s27,a23) (s28,a28) (s28,a23)}

Interdependence was created when each of students initiated and received *Choice* connections by selecting their work partners, and negotiating how they wanted to work on the project. For example, two students (s27 and s28) were found to be collaborating on a paper about the Olympics (a23). This is represented by the ordered pairs (s27, a23) and (s28, a23). They also completed other individual *Works* during this time i.e. (s27, a27), (s28, a28).

The *Morning Work Period* was also characterized by the presence of more *Works* (a1 to a28) that could be chosen by students, resulting in a larger *Size* (number of components in a system). As a result, the system showed more *Heterarchical Orderness* as *Choice* relationships were not prescribed hierarchically from one source, i.e. the teacher.

Structural Configurations With Respect to Instructional Affect Relations

From the analysis of classroom interactions, it was found that *Instructional* affect relations were created between teachers and students when they exchanged instructional content. This could occur through lectures and demonstrations, seeking and giving of instructional help or during project feedback sessions. It could also occur between students when they sought and received instructional help from each other to complete their *Works*. Differences in *Instructional* affect relations were found during three time periods: before starting *Head Problems*, during *Head Problems* and during the *Morning Work Period*.

Before Starting Head Problems.

Direct instruction was the primary mode of instruction used before teachers distributed *Head Problems* worksheets for the day. Children would be gathered at one side of the classroom where the requirements of the worksheet were reviewed with a short refresher of related contents. This usually lasted for about 15 minutes. This is represented by the affect relation set:

$$\{(t1,s1) (t1,s2) (t1,s3) (t1,s4) (t1,s5) (t1,s6) (t1,s7) (t1,s8) (t1,s9) (t1,s10) (t1,s11) (t1,s12) \\ (t1,s13) (t1,s14) (t1,s15) (t1,s16) (t1,s17) (t1,s18) (t1,s19) (t1,s20) (t1,s21) (t1,s22) (t1,s23) \\ (t1,s24) (t1,s25) (t1,s26) (t1,s27) (t1,s28) \\ (t2,s1) (t2,s2) (t2,s3) (t2,s4) (t2,s5) (t2,s6) (t2,s7) (t2,s8) (t2,s9) (t2,s10) (t2,s11) (t2,s12) \\ (t2,s13) (t2,s14) (t2,s15) (t2,s16) (t2,s17) (t2,s18) (t2,s19) (t2,s20) (t2,s21) (t2,s22) (t2,s23) \\ (t2,s24) (t2,s25) (t2,s26) (t2,s27) (t2,s28) \\ (t3,s1) (t3,s2) (t3,s3) (t3,s4) (t3,s5) (t3,s6) (t3,s7) (t3,s8) (t3,s9) (t3,s10) (t3,s11) (t3,s12) \\ (t3,s13) (t3,s14) (t3,s15) (t3,s16) (t3,s17) (t3,s18) (t3,s19) (t3,s20) (t3,s21) (t3,s22) (t3,s23) \\ (t3,s24) (t3,s25) (t3,s26) (t3,s27) (t3,s28) \\ (s1,t1) (s2,t1) (s3,t1) (s4,t1) (s5,t1) (s6,t2) (s7,t2)\}$$

When direct instruction was used, *Instructional* affect relations were initiated from the teacher to each student resulting in a high level of *Centrality* (see Figure 3). This was because the teacher was primary-initiating i.e. where they initiated rather than received these relations. Despite the high level of *Centrality*, there was a corresponding presence of *Complete Connectivity* and *Interdependence* because teachers always called upon students to share their knowledge, and used their answers to reinforce important concepts. Comparison of observation data during this time found that about 5 to 7 students were called upon during these mini-lecture

sessions. This is modeled with ordered pairs (s1,t1) (s2,t1) (s3,t1) (s4,t1) (s5,t1) (s6,t2) (s7,t2) in the affect relation set. By doing so, teachers allowed *Instructional* affect relations to be initiated and received with students; as exemplified by how one teacher explained a Math concept:

Teacher: Can someone remind us how we can calculate the area? (Students raise hands and teachers call upon a student.)

S1: It's length times the width

Teacher: Yes. If it's a square, you square the length. Then what is the perimeter, S2?

S2: It's the edge of a square.

Teacher: I'd say the perimeter is an ant's vacation. It's the distance walked during the vacation.

S2: You add the length of all the sides together.

Teacher: Or, you can take a short cut with the rectangle. If both the long side and short side are the same length, you take 2 times what? S3?

S3: Two times the length + two times the width.

During Head Problems.

Work-time during *Head Problems* was structurally similar to that for the *Morning Work Period* except that students had no choice over their learning activity. This is modeled by the affect relation set:

{(a100,s1) (a100,s2) (a100,s3) (a100,s4) (a100,s5) (a100,s6) (a100,s7) (a100,s8) (a100,s9)
 (a100,s10) (a100,s11) (a100,s12) (a100,s13) (a100,s14) (a100,s15) (a100,s16) (a100,s17)
 (a100,s18) (a100,s19) (a100,s20) (a100,s21) (a100,s22) (a100,s23) (a100,s24) (a100,s25)
 (a100,s26) (a100,s27) (a100,s28)}

(s21,t2) (t2,s21) (s22,t2) (t2,s22)

(s1,t3) (t3,s1) (s2,t3) (t3,s2) (s28,t3) (t3,s28) (s28,s10)

(s18,s19) (s19,s18)

(s4,t1) (s4,t2) (s4,t3) (s4,computer1) (s4,s1) (s4,s2) (s4,s3) (s4,s5) (s4,s6) (s4,s7) (s4,s8)

(s4,s9) (s4,s10) (s4,s11) (s4,s12) (s4,s13) (s4,s14) (s4,s15) (s4,s16) (s4,s17) (s4,s18) (s4,s19)

(s4,s20) (s4,s21) (s4,s22) (s4,s23) (s4,s24) (s4,s25) (s4,s26) (s4,s27) (s4,s28)}

Centrality was also higher as compared to the *Morning Work Period* (See Figure 3) since a larger number of *Instructional* affect relations originated from *Head Problems* assignments, as shown by ordered pairs (a100,s1), (a100,s2)...(a100,s28) in the affect relation set.

Complete Connectivity arose out of *Instructional* affect relations that were formed when students sought help from teachers and peers to solve *Head Problems*. It was observed that teachers rarely told students answers directly but used this time for personalized coaching. This creates bi-directional *Instructional* affect relations between teachers and students, as modeled by ordered pairs such as (s21,t2), (t2,s21) and (s1,t3), (t3,s1) in the affect relation set. A teacher shared how this was typically carried out:

I'll ask them what they know and have them reflect on other problems that they have previously done. That usually should coax an answer out of them. If not, I'll remind them of an operation or a function or let them work on a few problems with me. I could even direct them to relevant classroom resources.

The same strategy was applied even when students asked factual questions such as the correct spelling of a word. Rather than giving a direct answer, the teacher handed a dictionary to

the student, and stood by ready to help should he have difficulty. Personalized coaching was a means for creating *Complete Connectivity* between teachers and students as learning guidance was initiated to encourage students to share their existing knowledge; which in turn created opportunities for teachers to initiate more learning guidance.

Complete Connectivity was also observed between students. Even though each of them was allocated a work desk, they were free to move around and work with other students if they wanted to. At one end of the classroom, small groups of two or three could be gathered around the teachers' tables clarifying a question about the *Head Problems*. This is modeled by ordered pairs (s1,t3) (t3,s1) (s2,t3) (t3,s2) (s28,t3) (t3,s28) in the affect relation set where s1,s2, and s28 initiated *Instructional* affect relations with t3. S28 then returned to her desks and instructed another, as modeled by (s28,s10), where s28 forms *Instructional* affect relations with s10 by explaining what she learned from discussion with the teacher. At another work table, a student was seen helping another check the steps of his calculations for a Math problem, as represented by ordered pairs (s18, s19) (s19, s18) in the affect relation set.

Teachers also created opportunities for *Interdependence* by encouraging students to raise questions about the assigned *Head Problems*, especially when they found mistakes. For example:

Teacher: Now guys, listen up. S1 has a good question about Problem 5.

S1: It says 2.5 before 12:30p.m. It is 2.5 what?

(Four students gave suggestions): Minutes? Seconds?

Teacher: Ok – let's make it hours.

Sometimes, issues with the *Head Problems* could only be resolved with some additional research. For example, a problem stated that: "A Sunkist soda contains X amount of caffeine. How many Hershey bars have the same amount of caffeine?" A student (s4) highlighted that

information about the amount of caffeine in a Hershey bar was missing, thereby initiating *Instructional* affect relations with teachers as represented by the ordered pairs (s4,t1), (s4,t2), and (s4,t3) in the affect relation set. Teachers created opportunities for *Interdependence* between students by assigning s4 to be responsible for researching the information on the World Wide Web, and disseminating it to the class. *Instructional* affect relations were created with a classroom resource as represented by (s4,computer1) and also between s4 and other students (e.g. (s4,s1)...(s4,s28) when the required information was being shared.

Teachers believed that this strategy motivated buy-in for undertaking challenging work as it provided opportunities for students to initiate suggestions for improving the contents they were working with. A teacher commented:

Children have to put themselves on the line to try some assignments, which may be hard for them. They are encouraged to spot errors in the *Head Problems* I write on a weekly basis. Since we “criticize” them this much by the feedback we give them in the assignments, they should be able to “criticize” adults too. I see this as a chance to let them come back to me.

Modeling by teachers also encouraged students. When the missing value for the Hershey bar problem was found, one teacher asked if there was the need to consider the size of the chocolate bar. This immediately sparked further questions from students about the need to consider the type of chocolate used when applying the figure.

Morning Work Period.

At first sight, the Morning Work Period could seem unstructured or even chaotic as each teacher and student seemed to be engaged in their own agendas. Five or six students could be typing up book reports on the computers while two or three others are working on Math

workbooks at their desks. One teacher could be searching for a book with two students in the library; the second engaged in an individual feedback session while the third was grading assignments at her desk. In one corner, four or five students may be observing the growth process of caterpillars bred by a teacher while pairs of students may be working together on Science experiments at another corner.

A significant difference between *Head Problems* and the *Morning Work Period* was that *Instructional* affect relations with students originated from the different *Works* they chose to be engaged in resulting in a much lower value for *Centrality* as compared to the other two time periods. This is modeled by (a1,s1)...(a28, s28) in the *Instructional* affect relation set for the *Morning Work Period*:

{(a1,s1) (a2,s2) (a3,s3) (a4,s4) (a5,s5) (a5,s7) (a6,s6) (a8,s8) (a9,s9) (a10,s10) (a11,s11)
 (a12,s12) (a13,s13) (a14,s14) (a15,s15) (a16,s16) (a17,s17) (a18,s18) (a19,s19) (a20,s20)
 (a21,s21) (a22,s22) (a23,s23) (a24,s24) (a25,s25) (a26,s26) (a27,s27) (a28,s28)}

(computer1,s12) (s12,computer1) (computer2,s13) (s13,computer2) (computer3,s14)
 (s14,computer3) (computer4,s15) (s15,computer4) (computer5,s16) (s16,computer5)
 (computer6,s6) (s6,computer6) (s27, computer7) (computer7, s27) (s28, computer7)
 (computer7, s28) (books1,s14) (s14, books1) (books2,s20) (s20,books2)

(t1,s1) (s1,t1) (t1,s2) (s2,t1) (t1,s3) (s3,t1)

(t2,s6) (s6,t2) (t2,s9) (s9,t2)

(t3,s19) (s19,t3) (t3,s20) (s20,t3) (t3,s21) (s21,t3)

(s4,s5) (s5,s4) (s27,s28) (s28,s27)}

Size was also larger during the *Morning Work Period* (see Figure 3) because individual *Works* and resources such as the World Wide Web, drill-and-practice software, and reference books provide content information to support student learning. These are modeled by ordered pairs such as (computer1, s12) (s12, computer1), and (books1, s14) (s14, books1). Bi-directional *Instructional* affect relations were created as students searched for and received content information from these resources.

The *Morning Work Period* was also a time where teachers met individually with students to provide feedback on their projects. These interactions are illustrated by the bi-directional connections occurring between teachers and students such as (t1,s1) (s1,t1). Each project was given a separate grade for quality of contents, mechanics, number of errors made, and the number of attempts made to arrive at the *Last Draft* - that which was considered satisfactory by teachers. The feedback process was important time for personalized instruction and teachers gave much emphasis to it. In fact, it was described by one teacher as a time where “most of the teaching happens.” A meeting could sometimes take up to 45 minutes where teachers not only clarified misconceptions but also provided support by recommending books and resources that help students improve their projects. This thorough feedback process contributed to *Complete Connectivity* and *Interdependence* between teachers and students in terms of *Instructional* affect relations.

The flexibility for students to engage in collaborative projects also created opportunities for *Interdependence* in *Instructional* affect relations during this time. For example, s4 and s5 chose to help each other check their drafts for a Science project on the topic of Convection. They were observed to be asking each other questions related to their drafts, thereby forming bi-

directional *Instructional* affect relations as shown by (s4,s5) (s5,s4). At a computer station, s27 and s28 were found to be looking up information on the World Wide Web for a writing project on the Olympics, thereby create *Instructional* affect relations between themselves and computer resources.

Despite increased opportunities for collaboration, *Interdependence* during the *Morning Work Period* was lower than the other two time periods because it was left to the sporadic intent of students. This meant that affect relations were formed only between those students who chose to collaborate. In comparison, *Interdependence* during *Head Problems* usually involved instructional information being shared with the whole class thereby resulting in relationships being formed automatically between each student. This also accounted for the fact that the *Morning Work Period* had the lowest value for *Complexity*.

Structural Configurations with Respect to Support Affect Relations

Even though *Instructional* affect relations were predominant, *Support* affect relations were also found to coexist with them. These involved interactions related to information that was required to support instruction but were not instructional in nature. During *Head Problems*, such interactions usually involved the clarification of work instructions. For example, the teacher gave students more questions on fractions than they would normally get for *Head Problems* one day:

S1: There are 16 fractions in the worksheet.

Teacher: I see only four.

S2: There's more at the back of the sheet.

Teacher: Oh there it is! S3, why don't you take a look at these and determine which set is better for the 4th graders?

S4: Let's do all of them!

Teacher: I want to take a look and see which are harder. We can count these as questions with bonus points.

This is modeled by the following affect relation set:

{(s1,t1) (s2,t1) (s4, t1) (t1,s1) (t1,s2) (t1,s3) (t1,s4) (t1,s5) (t1,s6) (t1,s7) (t1,s8) (t1,s9)
 (t1,s10) (t1,s11) (t1,s12) (t1,s13) (t1,s14) (t1,s15) (t1,s16) (t1,s17) (t1,s18) (t1,s19) (t1,s20)
 (t1,s21) (t1,s22) (t1,s23) (t1,s24) (t1,s25) (t1,s26) (t1,s27) (t1,s28) }

By allowing student input e.g. ordered pairs (s2,t1) and (s4,t1), teachers created opportunities for *Complete Connectivity* and *Interdependence* in terms of *Support* affect relations. However, such incidences during *Head Problems* were sporadic, resulting in a lower value for *Complete Connectivity*, *Interdependence* and *Strongness* (the degree to which components are connected) (See Figure 4).

In comparison, *Support* affect relations were more predominant during the *Morning Work Period* as students were not constrained to doing the same activity at the same time. This provided greater flexibility for students to engage in *Support* activities with each other.

For example, two students taught another how to use the scanner and touch up a graphic:

S1: I need help scanning.

S2: I know – keep pressing Scan.

S3: No. Don't go to anything yet. OK – press Scan now. Oh, you didn't put the picture.

Let me help you.

S1: Nothing is appearing.

S3: It's scanning – why is it taking so long?

(They see a dialog box on the computer screen and decided that the scanner was working effectively on the job)

S2: Ok – now how do I color the picture?

S3: You press Color (demonstrates how to do it)

S2: Oh! Look at this!

This is modeled by the interactions between s1, s2, s3 and the scanner in the *Support* affect relation set during the *Morning Work Period*:

{(s1,s2) (s3,s1) (s1,s3) (s3, scanner), (scanner,s1)}

(s10,s11) (s11, s10)

(s23, s24) (s24,s23)

(t1,s26) (t1,books4) (s26,t1) (s26,books4)

(t2,s25) (s25,t2)

(s7,books1) (s8,books2) (s14,computer1) (s15,computer2) (s16,computer3) (s17,computer4)

(computer1,s14) (computer2,s15) (computer3,s16) (computer4,s17) (books1,s7) (books2,s8)}

Another type of *Support* affect relation that occurred naturally between students would be during the orientation of new students. For example, when approached by new student s11, s10 demonstrated how to clip a grading sheet to the *Head Problems* worksheet when handing it up. S23 served as a student mentor for s24 who was a visitor from the lower elementary class by showing her the format for citing sources in a research report. Teachers also provided support to

students by helping them search for additional books and resources to support completion of Works as shown by interactions between t1, s26 and books4.

Besides obtaining support from teachers and peers, students were also able to initiate *Support* affect relations with learning resources such as computers, scanners, transparencies, reference books and experimental apparatus that were needed to complete their projects. As a result, *Complete Connectivity*, *Interdependence* and *Strongness* were higher during the *Morning Work Period* as there was a larger number and more variety in the types of *Support* relations that were present.

Structural Configuration with Respect to Control Affect Relations

Even though student autonomy was a feature of the Montessori classroom, observations showed that this did not preclude the need for control. When asked how they determined if control was necessary, a teacher shared this principle, “When you have disorganized kids, it is time for control. Certain kids are more disorganized, and when their behavior becomes dominating and contagious, control is necessary. Examples of disorganized behavior are walking around and being noisy.”

Figure 5 shows that *Control* affect relations were characterized by the presence of *Hierarchical Orderness* and *Passive Dependence*, which described connections whereby components received, rather than initiated relationships. *Heterarchical Orderness* and *Interdependence* were absent as *Control* was imposed on students without any means for negotiation. This is modeled by the affect relation set:

$$\{(t1,s1) (t1,s2) (t1,s3) (t1,s4) (t1,s5) (t1,s6) (t1,s7) (t1,s8) (t1,s9) (t1,s10) (t1,s11) (t1,s12) \\ (t1,s13) (t1,s14) (t1,s15) (t1,s16) (t1,s17) (t1,s18) (t1,s19) (t1,s20) (t1,s21) (t1,s22) (t1,s23) \\ (t1,s24) (t1,s25) (t1,s26) (t1,s27) (t1,s28)\}$$

Observations showed that teachers initiated *Control* affect relations in four situations. One was to focus students for instructional purposes; for example when teachers asked students to pay attention to a transparency used to explain Math concepts that were relevant for *Head Problems*. The second type of control was for disciplinary reasons. An issue arose where students left the cleanup duties for the previous day uncompleted. Teachers decided that students would not be allowed to work on the computers until their assigned jobs were completed. The aim was to help them understand that all duties for a day should be completed before the next started. The third type of control occurred when teachers were trying to help students stay on task during the *Morning Work Period*. For example, when a pair of parakeets that were bred in the classroom were let out of their cages for some exercise in the morning, some students who have completed their *Head Problems* attempted to “bring” them to their desks. Momentarily disruptions to the order of the classroom occurred and the teacher found it necessary to institute control as follows:

I ask that no one put the bird on your sleeve and bring him around. It causes great confusion in the class, not to say that it may distress the bird. If you’re fortunate that he choose to come by you and stay, that’s great.

The fourth example of control was to help students transition between *Head Problems* and the *Morning Work Period*. Even though students were given an hour to complete *Head Problems*, teachers were not insistent that those who finished early proceed immediately to their *Works* as they acknowledged that students needed “important social time” with each other. It was not uncommon to find some students taking advantage of this. About 15 minutes before the official start time for the *Morning Work Period*, a pair at a work table shared a story while doodling up a cartoon character on some paper. Those presenting their answers to the *Head*

Problems wrote their answers on a transparency over some small talk. A few gathered round a teacher and asked when she would expect the caterpillars being bred in the class to turn into butterflies while another group stood by the parakeet cage and laughed at the birds' antics. However, when it came time for the *Morning Work Period*, it was common for teachers to issue express orders for students to get settled down to work if they did not do it automatically.

Impact on Student Motivation

The *Academic Self-Regulation Survey* was completed by nine students. Table 1 shows the average rating of respondents by motivation category. Deci, Eghrari, Patrick and Leone (1994) proposed internalization as the process whereby people proactively assimilated and integrated behaviors regulated by external contingencies. External Regulation referred to behaviors that were executed only if there was initiation by external conditions while Introjected Regulation referred to behaviors that were not initiated by external conditions, but were exhibited unwillingly as they were not regarded as part of the integrated self. When people executed behaviors because they valued, identified and accepted them as part of self to some extent, they exhibited Identified Regulation.

For both surveys, average rating of respondents was highest for Identified Regulation, followed by Introjected Regulation, External Regulation and Intrinsic Motivation. These figures showed that respondents had a greater tendency to undertake learning activities because they perceived some personal value and identification with the learning goals rather than because they felt compelled by external factors. Given the emphasis on independence and autonomy in a Montessori system, it seemed contradictory that Intrinsic Motivation would be rated lower than External Regulation. When asked about this, a teacher commented:

The results don't surprise me. I've tried using intrinsic rewards to motivate kids. Some kids are great under the laissez-faire system, but many kids get lost. Across time, I've instituted the punch card system where they can check how they are progressing. The use of this system might not give them the feeling that they are doing it for themselves.

Since about 90% of students in the class spent at least three years at this Montessori school, a second teacher added that:

Another reason could be that most of our kids have never been in any other types of academic settings. When they are 10 or 12, they don't think they need adult input to do their work. But, they may not be aware of how much freedom they are having in our system.

Motivation Styles of Teachers

The teachers' score on the *Problems in Schools Questionnaire* by motivational style is shown in Table 2. The maximum possible score for each category was 56. The three teachers varied in terms of their teaching experience in Montessori schools. The Head Teacher had 32 years of teaching experience, while the two assistant teachers had 4 and 1 year of teaching experience respectively. Regardless of the number of years they taught in a Montessori school, all the three teachers had the highest scores for the category related to *Highly Autonomy Supportive* and lowest scores for the *Highly Controlling* category. When given scenarios asking for how they would handle motivational problems with school children, all teachers chose the use of highly autonomy-supportive strategies such as communication to understand the perspective of the child, and encouragement rather than highly controlling strategies such as extrinsic rewards and punishment. This was attested by a teacher who said that, "We don't really

use extrinsic rewards in the Montessori system. Maybe the closest we get is making bread, because to qualify for making bread, you need to be caught up in your work.”

The use of these strategies was also observed in how teachers handled similar situations in their classroom. For example, during the incident where students did not complete assigned duties for clean-up, teachers did not directly reprimand or punish students even though they knew who were responsible. Instead, the problem was opened up for discussion with the class where one teacher said:

As a general policy, we do not end the day until all the jobs are done. If you have problems doing this, we can discuss and come up with a plan. We cannot leave our classroom like trash so that you can socialize. What can we do?

Students who were responsible immediately apologized to the class, and quickly proceeded on with the unfinished tasks. As a warning, the teachers did not allow other students to work on computers until those students got done. No overt blame from other students was noticed except that some waited anxiously by the computer terminal they wanted to use. A loud cheer from the class was heard when those on duty pronounced them completed. Students rushed quickly to continue their *Works* on the computers.

Discussion

The results of this study show that the instructional methods employed by teachers were congruent with the basic Montessori philosophy of cultivating self-mastery. Regardless of whether students sought help for *Instructional* or *Support* issues, teachers consistently used these opportunities to help them arrive at their own answers for resolving problems. This resulted in the presence of *Complete Connectivity* throughout *Head Problems* and the *Morning Work Period* in terms of *Instructional* and *Support* affect relations. This underlying philosophy for Montessori

teaching accounts for why all the three teachers rated themselves as being *Highly Autonomy Supportive*.

Studies by Reeve *et al.* (1999) and Manouchehri (2004) found that autonomy-supportive teachers listened more, gave fewer directives and asked more questions about what students wanted to do, and were more responsive to students' questions. This study shows that *Complete Connectivity* in terms of *Instructional* affect relations could also be created when teachers provide opportunities for students to contribute ideas and suggestions for improving the learning contents they are working with. This recognizes students as legitimate partners in the learning process, and impacts their willingness to learn challenging content. In this situation, even prescribed learning activities could have a high level of personal relevance for students.

In terms of *Support* affect relations, a free-flowing work system such as that of the *Morning Work Period* provides scope for *Complete Connectivity* and *Interdependence* between students. When *Support* affect relations are less centralized and dependent upon the availability of teachers, it allows them to focus on personalized instruction. The structural organization of the *Morning Work Period* also increases the *Strongness* of connections between system components, providing opportunities for students to foster social relationships and relatedness, or secure connections with people (Stipek, 2002). This was identified by Self-determination theory as another basic human need which could contribute to the development of intrinsic motivation (Deci *et al.*, 1999). Teachers can stimulate the formation of relatedness by legitimizing social-time between students as part of classroom practices.

While Assor *et al.* (2005)'s study found that controlling strategies predicted higher levels of anger and anxiety, this study found that being autonomy-supportive does not imply the absence of control. On the contrary, discipline and control are periodic and necessary activities

for maintaining engagement on learning goals. They need not necessarily result in negative affect with students if used appropriately.

Analysis of Patterns in Time and Configuration (APT&C).

The measures of classroom structure reported in this study are new. A little background may be useful for the reader to understand their development. Originally, Frick (1990) devised analysis of patterns in time (APT). APT has since been extended by Thompson (2006) to include analysis of patterns in configurations (APC). Together, this approach to analysis and measurement is called APT&C.

Analysis of patterns in time (APT) was developed as a methodology for measuring system dynamics – i.e., temporal configurations or processes (Frick, 1983; 1990). APT was, and still is, a paradigm shift in thinking for quantitative methodologists steeped in the linear models tradition and the measurement theory it depends on. The fundamental difference is that *the linear models approach relates independent measures through a mathematical function and treats deviation as error variance, but APT measures a relation directly by counting occurrences of when a temporal pattern is true or false in observational data.* Linear models relate the measures; APT measures the relation.

APT is well explicated in Frick (1990; 1983). As an example of conclusions when using APT: In an observational study of mildly handicapped children in elementary school Frick (1990) found that, regardless of classroom context, when direct instruction was occurring these students were engaged on average about 97 percent of the time. In the absence of direct instruction, their engagement was about 57 percent. In other words, such students were 13 times more likely to be off-task during non-direct instruction.

This kind of APT finding is similar to epidemiological findings in medicine. For example, heavy cigarette smokers are 5-10 times more likely to have lung cancer later in their lives (Kumar, et al., 2005), and if they quit smoking the likelihood decreases. While causal conclusions cannot be made in the absence of controlled experiments, nonetheless one can make practical decisions based on such epidemiological evidence. We can do likewise with APT.

A very recent example of APT further illustrates the utility of this approach in a study of theoretically-based course evaluation items. In this investigation, Frick, Chadha, Watson, Wang and Green (2007) reported that:

... analysis of patterns in time indicated that students were 3-5 times more likely to learn a lot and were satisfied with courses when first Principles of Instruction were used *and* students were frequently engaged successfully [Academic Learning Time (ALT)].

Students were 9 times more likely to master course objectives when both First Principles and ALT occurred, compared with their absence. (p. 34)

Thompson (2006) concluded that APT could be extended to characterize structure or configuration of educational systems. Configural patterns characterize *structures* in education – i.e., how education is organized, or relations between parts and whole. Thus, working together, Thompson and Frick have since been developing APT&C. The foundation for this development is ATIS (Axiomatic Theories of Intentional Systems). ATIS, in turn, has built on concepts from graph theory (cf. Thompson, 2006) and the SIGGS Theory Model (Maccia & Maccia, 1966). These structural properties include: size, complexity, active dependence, passive dependence, centrality, compactness, complete connectedness, flexibleness, heterarchical orderness, hierarchical orderness, independence, interdependence, strongness, unilateralness, vulnerableness, weakness, and

wholeness. Frick has recently developed a preliminary version of software for analysis of patterns in configurations (APC) based on measures defined by Thompson (2006).

Further software is being developed for APT, building upon an earlier version from the 1980s (see Frick, 2005; Frick, An & Koh, 2006).

This methodology for measuring both dynamics and structure is referred to as APT&C. The present study used the APC software for computing the results reported herein, and is the first research of which we are aware that has employed this approach to measuring configurations of educational learning environments and systems. The value of this approach is illustrated by characterizing and measuring structures in a Montessori classroom.¹

Limitations and Suggestions for Future Research

The first limitation of this study is that only one Montessori classroom was studied. Comparison and contrast of structural configurations between systems is not possible, thereby limiting the generalization of results to other Montessori classrooms. The second limitation is that only about a third of the students could be surveyed. The motivational profiles of students may not have been represented accurately. Therefore, the student survey results need to be interpreted with caution. It was also not possible to make comparisons with K-12 classrooms as no previous studies have been conducted using the system properties defined by ATIS. The measures of system properties should not be used as benchmarks to assess the level of autonomy-support in classrooms.

For future research, this study could be replicated in more Montessori and K-12 classrooms and the structural properties compared. The measures of *Complete Connectivity* with respect to *Instructional* and *Support* affect relations could be analyzed further to determine its

¹ APT was not utilized in the present study.

impact on autonomy-support and intrinsic motivation. Future research could also explore the relevance of other structural systems properties such as *Compactness*, *Flexibleness*, *Weakness* and *Vulnerableness* towards explaining classroom configurations. This would provide allow additional dimensions of classroom behavior to be understood.

Conclusion

This study investigated nine system properties that are relevant for describing the structural configurations of Montessori classrooms. Attainment of self-mastery and intrinsic motivation for learning in a Montessori classroom is facilitated through the use of autonomy-supportive strategies that increase *Complete Connectivity* and *Interdependence* with respect to *Instructional* and *Support* affect relations. Autonomy support must also be implemented with an aim to focus students on learning goals. To this end, *Control*, when used appropriately, is necessary.

Analysis of patterns in configurations (APC) provides a rigorous theoretical and logico-mathematical foundation for a *new approach* to measuring classroom structure that is part of Axiomatic Theories of Intentional Systems (ATIS). APC gives us a new language and new way to understand and compare classrooms and learning environments. Structural properties of different kinds of learning environments or approaches to education can be measured with APC, compared, and ultimately related to important learning outcomes. The study reported here is just a beginning. Nonetheless, results clearly show how APC measures can be used to characterize autonomy-supportive strategies.

Even more important is what the ATIS theory predicts for a given system under specific conditions. For example, if strongness of affect relations increases, the following axioms in ATIS are relevant for that particular system:

055: If *strongness increases*, then hierarchical order decreases.

056: If *strongness increases*, then flexibility increases.

106: If *strongness increases*, then toput increases.

107: If *strongness increases*, then input increases.

108: If *strongness increases*, then filtration decreases.

144: If filtration decreases, then isomorphism increases.

151: If isomorphism increases; then fromput decreases, and feedout decreases.

005: If toput increases, then input increases to some value and then decreases.

008: If toput increases, then filtration decreases to some value and then increases.

009: If toput increases, then regulation less than some value increases.

090: If toput increases, then centrality decreases.

While beyond the scope of this report, it can be seen that 11 ATIS axioms are triggered when the system property strongness increases. Strongness measures would be computed for each affect relation set within the system and then averaged, according to the definition of the strongness measure earlier in this report. If the average amount of strongness increases, then ATIS predicts other changes in the system. Notice, for example, that both input and toput are predicted to increase if strongness increases. ATIS input and toput are further system properties. ATIS predicts changes we would not expect from self-determination theory. ATIS has the potential to help us better understand and predict the behavior of educational systems. More information on ATIS is available in reports by Thompson (2005a; 2005b; 2005c; 2006; 2007).

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Table 1.

Students' Motivation Level for Schoolwork

Type of motivation	Score (Beginning)	Score (End)
External Regulation	2.47	2.54
Introjected Regulation	3.04	2.88
Identified Regulation	3.13	3.12
Intrinsic Motivation	2.15	2.10

Note. n = 9

Table 2.

Teachers' Motivational Style

Motivational Style	Average Score
Highly autonomy supportive	46.33
Moderately autonomy supportive	28.67
Moderately controlling	23.00
Highly controlling	16.00

Note. n = 3

Figure 1. Matrix of the *Instructional* affect relation set: $\{(s1,t2), (t2,s1)\}$

Received by

	s1	t2
s1	-	1
t2	1	-

Initiated by

Figure 2. Comparison of structural configurations with respect to student choice of learning activity

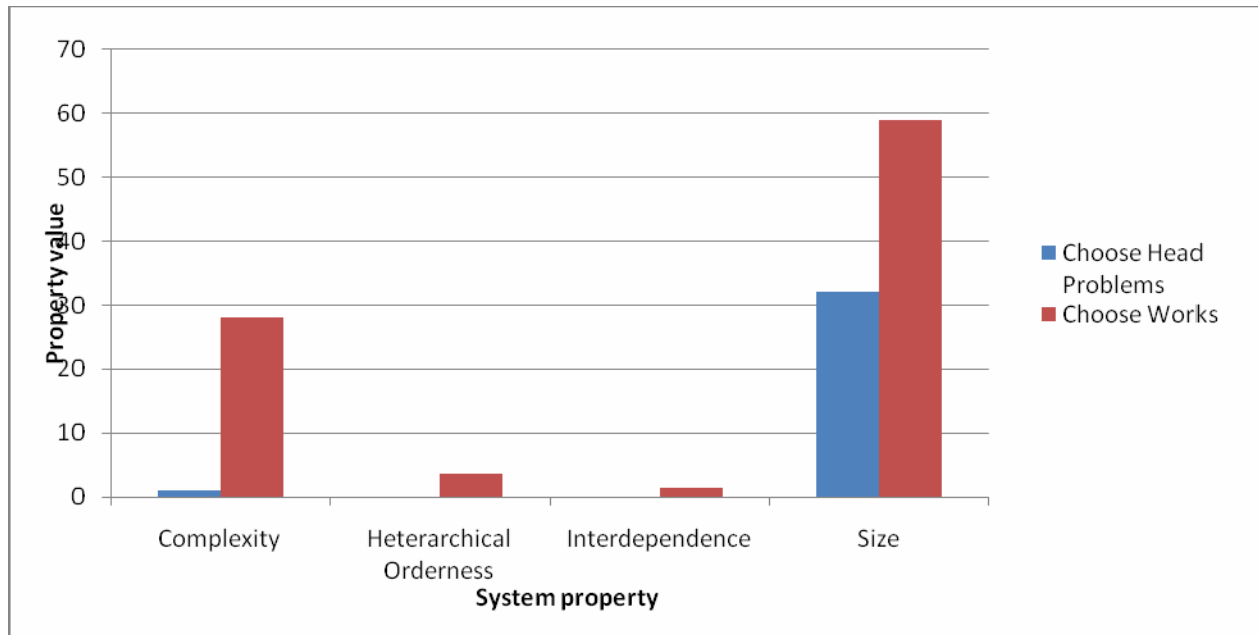


Figure 3. Comparison of structural configurations with respect to *instructional* relations

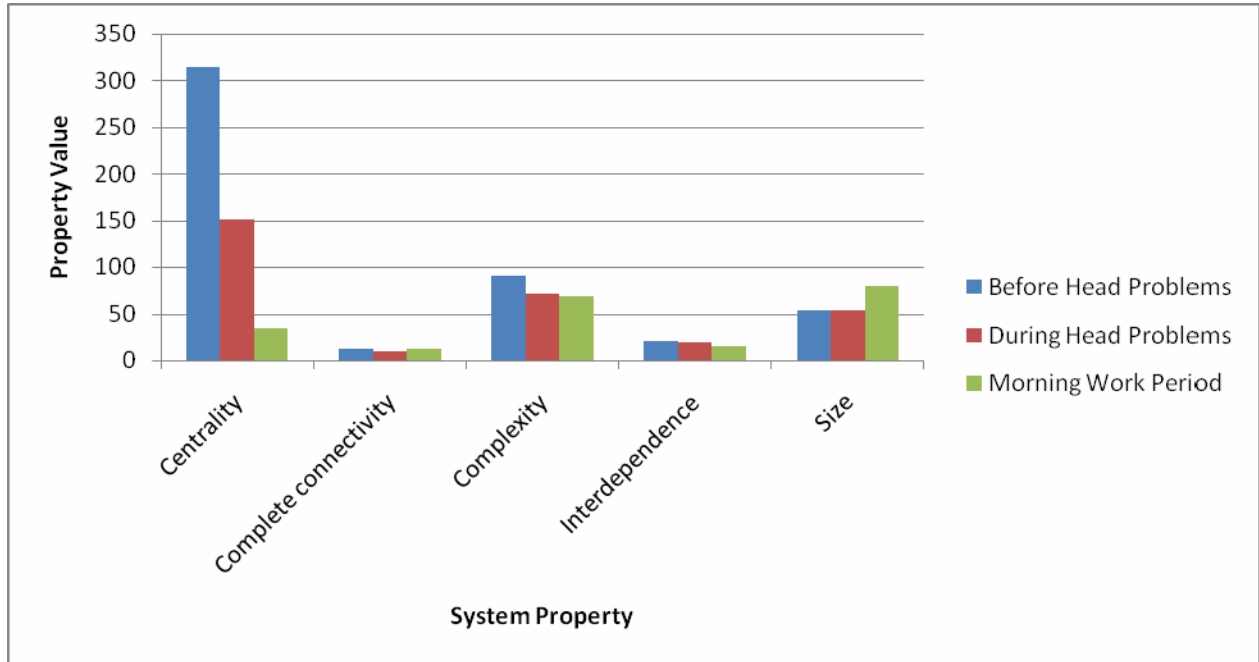


Figure 4. Comparison of structural configurations with respect to *support* relations

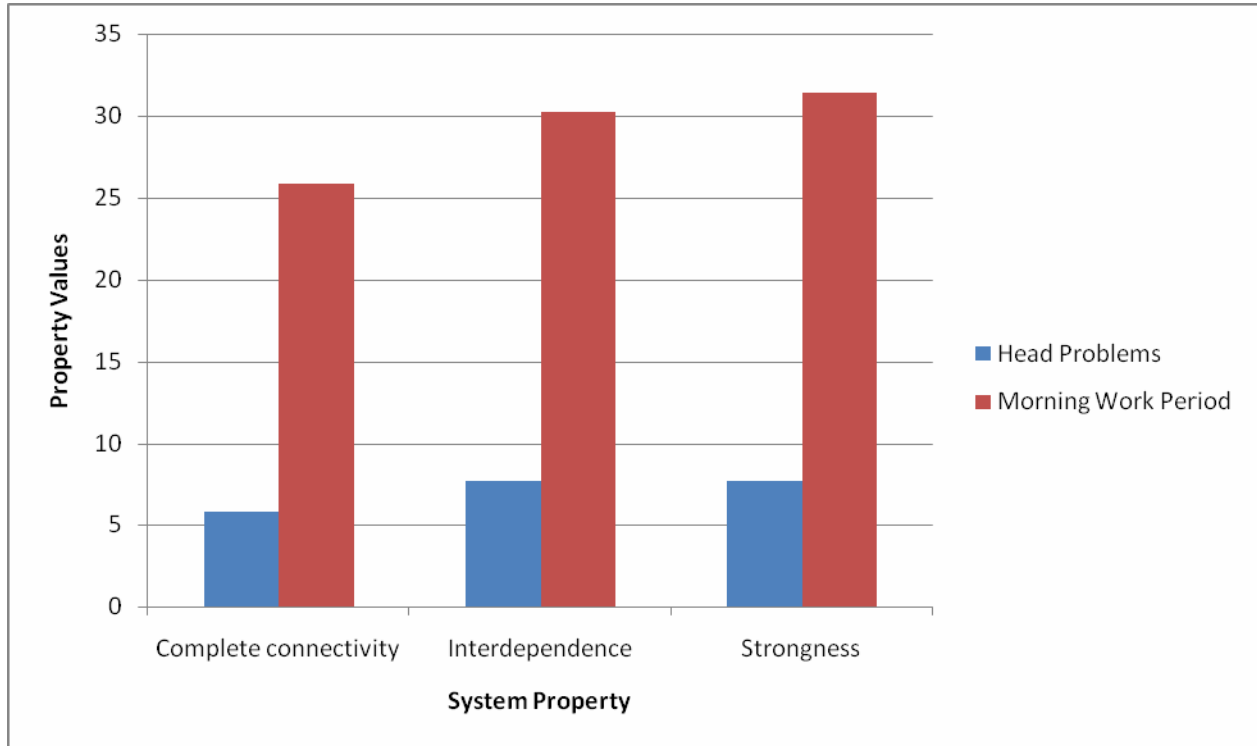


Figure 5. Structural properties relevant to *control* relations

